$c=[(t\mp q)^2-q^2]/2q$, and $d=t\mp q$; or, in integral form, $a=b=(t\mp q)^2+q^2$, $c=2t(t\mp 2q)$, and $d=4q(t\mp q)$.

Examples:—(1). Put $t=q=1$. Then $a=b=5$, $c=6$, and $d=8$.

(2). Put $t=3$ and $q=1$. Then $a=b=17$, $c=30$, and $d=16$. Also $a=b=5$, $c=6$, and $d=8$.

When $q=p$, we have, $a=[(3p+t)^2-p^2]/8p$, $b=[(3p+t)^2+7p^2]/8p$, $c=[(3p+t)^2-5p^2]/4p$, and $d=3p+t$.

When $t=q=p$, we have, in integral form, $a=15p$, $b=23p$, $c=22p$, and $d=32p$.

Thus we continue making general values for a, b, c, and d, under a number of other conditions; as, $t=q$; $t=p$; $t=2q=2p$, etc.

43. Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.

Find the series of integral numbers in which the sum of any two consecutive terms is the square of their difference.

I. Solution by J. H. DRUMMOND, LL. D., Portland, Maine, and the PROPOSER.

Let x and $x+m$ be two consecutive numbers. Then we have $2x+m=m^2$, and $x=m(m-1)/2$, and $x+m=m(m+1)/2$. But $m(m+1)/2$ is the sum of the terms in the series $1+2+3+4\ \ldots\ldots\ldots\ldots+m$. Hence the m^{th} term of the series required is the sum of m terms of this series, and we have 1, 3, 6, 10, 15, $\ldots\ldots\ldots\ldots m(m-1)/2$.

II. Solution by COOPER D. SCHMITT, M. A., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee; O. W. ANTHONY, M. Sc., Professor of Mathematics, New Windsor College, New Windsor, Maryland; and BENJ. F. YANNEY, A. M., Professor of Mathematics, Mount Union College, Alliance, Ohio.

By the conditions we must have $x+y=(x-y)^2$, x and y representing two consecutive terms in the series. Solving as a quadratic in x, we have $x=(2y+1)/2\pm\sqrt{(8y+1)/4}$. Hence $8y+1$ must be a square.

When $y=1$, $8y+1=3^2$, $x=3$;
$y=3$, $8y+1=5^2$, $x=6$;
$y=6$, $8y+1=7^2$, $x=10$;

and the series is, 1, 3, 6, 10, 15, 21, 28, 36, 45, etc., or the system of *triangular* numbers as set forth in Pascal's Triangle.

Also solved by *A. H. HOLMES*, *E. W. MORRELL*, *H. C. WILKES*, and *G. B. M. ZERR*.

44. Proposed by A. H. HOLMES, Box 963, Brunswick, Maine.

The hypotenuse of a right-angled triangle ABC, right-angled at A, is extended equally at both extremities so that $BE=CD$. Draw AD and AE. Find integral values for all the lines in the figure thus made.

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Construct the figure as indicated by the problem. Then draw BF equal and parallel to AC, and draw CF, AF, EF, and DF. Then will $ABFC$ be a rectangle; and the diagonals BC and AF are equal.

It is also evident that $AE=DF$ and $AD=EF$. Whence $AEFD$ is an ob-

lique-angled parallelogram, or rhomboid, of which AE and AD are the sides, and ED and AF the diagonals.

Let $x=BE=CD$, and put $a=AD$, $b=AE$, $c=ED$, and $d=AF=BC$, taking $c>d$. Then $2x+d=c$, and $x=(c-d)/2$. If $d>c$, AE and AD fall inside of AB and AC, and the hypotenuse BC would be *contracted* instead of *extended.*

We now find integral values for a, b, c, and d. This has been done in the solution of No. 42, in this issue, and need not be reproduced here.

By this process we find integral values for all the lines except the two legs, AB and AC, of the right-angled triangle. By means of the median and the perpendicular upon BC, we readily find

$$\overline{AB}^2=d[4b^2-(c-d)^2]/4c \text{ and } \overline{AC}^2=d[4a^2-(c-d)^2]/4c.$$

Now, if these expressions can be rendered squares, without destroying the relations of a, b, c, and d, AB and AC will also be rational and integral. But I have not yet succeeded in accomplishing this. We shall now illustrate by means of a few examples.

From Diophantine problem No. 42, take the set of values, $a=4$, $b=7$, $c=9$, and $d=7$. Then $2x+7=9$; whence $x=1$. $\therefore$ $AD=4$, $AE=7$, $ED=9$, $BC=AF=7$, $BE=DC=1$, $\overline{AB}^2=112/3$, and $\overline{AC}^2=35/3$.

Take the set of values, $a=8$, $b=11$, $c=17$, and $d=9$. Then $2x+9=17$; and $x=4$. Also $\overline{AB}^2=945/17$, and $\overline{AC}^2=432/17$.

Partial solutions also received from *J. H. DRUMMOND*, *A. H. BELL*, and the *PROPOSER*.

PROBLEMS.

51. Proposed by H. C. WILKES, Skull Run, West Virginia.

The difference between the roots of two successive triangular square numbers equals the sum of two successive integral numbers, the sum of whose squares will be a square number. Demonstrate.

52. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland.

Prove that a "magic square" of nine integral elements, whose rows, columns, and diagonals have a constant sum, is only possible when this sum is a multiple of three.